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THERMOCONVECTIVE WAVES IN A CONDUCTING AND RADIATING FLUID

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Abstract—The investigations deal with the influence of thermal radiation on the propagation of thermoconvective waves in a fluid heated from below. The radiation field is described by means of the differential approximation. The dispersion equation that is derived for horizontally propagating plane harmonic waves has three different solutions:

In addition to the two modes of thermoconvective waves one also obtains one wave of the radiation-induced type whereby interesting interactions are observed. If the radiation field leads to a strongly nonlinear temperature distribution in the undisturbed basic state the propagation of the weakly damped thermoconvective waves is described in terms of a ray theory.

NOMENCLATURE

Bu,	Bouguer-number;
b,	thickness of the layer;
c_p,	specific heat capacity at constant pressure;
e,	specific internal energy;
G,	parameter describing the relative importance of radiation, cf. equation (14);
$\mathbf{g}$,	gravitational acceleration, $\mathbf{g} = (0, -g)$;
H,	radiation–conduction parameter, cf. equations (13b) or (15);
I_ν,	spectral radiative intensity;
I_m,	mean radiative intensity, cf. equation (6);
K,	dimensionless parameter, cf. equation (28b);
k,	complex wave number, $k = k_r + ik_i$;
L_x,	characteristic length in x-direction;
N,	buoyancy frequency, cf. equation (18b);
$\mathscr{P}$,	phase of harmonic waves, cf. equation (31);
$\mathbf{P}$,	hydrodynamic stress tensor;
Pr,	Prandtl-number;
Q_0,	amplitude of external radiation flux, cf. equation (26);
$\mathbf{q}_R$,	radiative heat flux;
s,	specific entropy;
T,	temperature;
t,	time;
v,	velocity component in y-direction;
$\mathbf{w}$,	vector of flow velocity;
x,	horizontal coordinate;
y,	vertical coordinate.

Greek symbols

α,	radiative absorption coefficient;
β,	thermal expansivity;
γ_0,	stratification parameter, cf. equation (12);
ε,	frequency parameter, cf. equation (18b);
θ,	amplitude of temperature perturbation;
θ_0,	amplitude of oscillating wall temperature;
ϑ_r, ϑ_p,	inclination angles of ray direction and phase normal, respectively;
κ,	thermal diffusivity;
λ,	thermal conductivity;
μ,	viscosity;
μ_b,	bulk viscosity;
ν,	kinematic viscosity;
ξ,	ray coordinate;
ρ,	density;
σ,	Stefan–Boltzmann constant;
τ,	optical thickness, cf. equation (28b);
χ,	eikonal, cf. equations (30), (31);
ω,	angular frequency.

Subscripts, superscripts

0,	at dimensionless quantities: first order expansion; at non-dimensionless quantities: undisturbed state;
$'$,	perturbation quantity;
$\bar{}$,	dimensionless quantity;
$*$,	reference quantity;
x,y,t,	partial derivatives with respect to x,y,t, respectively.

1. INTRODUCTION

IT WAS shown at first by Luikov and Berkovsky [1] that in a stratified fluid in the gravity field the properties of coupled thermal and shearing waves may alter completely according to the influence of buoyancy effects. One has to deal with a new type of waves, so called thermoconvective waves. Most significant is the fact that the damping of the first mode of thermoconvective waves may be orders of magnitude smaller than that of classical thermal and shearing waves, whereas the simultaneously propagating second mode of thermoconvective waves is characterized by larger values of the damping. A detailed description of the phenomenon of thermoconvective waves is given in [3].

It has been shown [2, 3] that the buoyancy forces to produce thermoconvective waves must be at least of such a magnitude that they already give rise to

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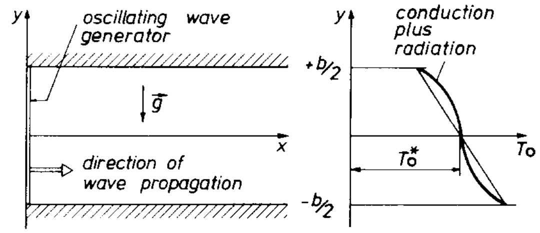

FIG. 1. Geometry of the problem: $T_0(y)$ temperature distribution in the undisturbed state; $\mathbf{g}$ vector of gravitational acceleration.

instability in an unbounded stratified fluid. At a large adverse temperature gradient as is necessary for thermoconvective waves, stability can only be guaranteed in a horizontal layer of finite thickness (Fig. 1) under the condition of a subcritical Rayleigh-number. Hence a treatment of thermoconvective waves in the form of one-dimensional plane waves as was performed in [1] seems to be inconsistent. The influence of bounding horizontal walls on the propagation of thermoconvective waves has therefore been analysed in [3]. It has been shown that under the condition of a large, yet subcritical Rayleigh-number ($Ra \gg 1$ with $Ra < Ra_{\text{crit}}$) the one-dimensional model, though disregarding the amplitude profile over the cross-section of the layer, actually yields a first approximation for the wave number. This result will also be most useful in the present analysis that is concerned with the propagation of thermoconvective waves under the influence of a radiation field.

Problems of radiation–convection interaction have attracted much interest during the last years, e.g [4, 5]; in these problems the relative importance of radiation may be surprisingly large already at moderate temperatures of about 500 K, cf. Section 2. On the other hand, the study of the influence of radiation on classical thermal waves [6] as well as on sound waves [7] led to the discovery of the phenomenon of radiation-induced waves. In connection with sound waves radiation-induced waves have already been verified experimentally, cf. [8].

The previous studies [6] about the simultaneous propagation of a thermal wave and a radiation-induced wave have shown remarkable and unusual results; e.g. at certain values of the parameters the two solutions of the characteristic equation may coincide forming a double-root or may exchange their physical meaning in the course of the variation of parameters. In the case of thermoconvective waves in a heat-conducting and radiating medium we have now to study the interaction of a radiation-induced wave not just with another single wave but with a system of two coupled waves, and we shall be able to make interesting comparisons between the behaviour of ordinary thermal waves and thermoconvective waves.

Radiation does not only influence the propagation of thermoconvective waves by means of the interaction with a radiation-induced wave but at the same time also by way of a nonlinear temperature distribution in the undisturbed basic state, see Fig. 1. In cases where a strong nonlinearity of the temperature profile does not allow a formulation in terms of plane waves the methods of the ray theory will be applied. In this context it has first to be shown that also under the influence of radiation the wave length of the weakly damped thermoconvective wave is small compared to distances over which the temperature gradient changes considerably.

A more detailed version of the analysis that will be outlined below can be found in the author's Ph.D. thesis [2].

2. GOVERNING EQUATIONS

The basic hydrodynamic equations representing the conservation of mass, momentum and energy are adopted in the following form:

$$\frac{D\rho}{Dt} + \rho \operatorname{div} \mathbf{w} = 0, \tag{1}$$

$$\rho \frac{D\mathbf{w}}{Dt} = \operatorname{div} \mathbf{P} + \rho \mathbf{g}, \tag{2}$$

$$\rho \frac{De}{Dt} = \mathbf{P} \colon \operatorname{grad} w + \operatorname{div}(\lambda \operatorname{grad} T) - \operatorname{div} \mathbf{q}_R. \tag{3}$$

$\mathbf{w}$ is the vector of the flow velocity with the components u, v in x, y-direction.

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \mathbf{w} \operatorname{grad}$$

denotes the substantial derivative with respect to time; ρ is the density, T the absolute temperature, e the specific internal energy, $\mathbf{g}$ the vector of gravitational acceleration, $\mathbf{P}$ the hydrodynamic stress tensor, λ the thermal conductivity. $\mathbf{q}_R$ is the radiative heat flux that couples the hydrodynamic and the radiative field. Radiation stresses and the radiant energy density are neglected, therefore excluding extremely low densities together with extremely high temperatures from our considerations, cf. e.g. [9].

If we supplemented the hydrodynamic equations (1)–(3) by the exact equations of the radiation field in terms of the transport theory we would have to deal with a system of integro-differential equations. It can be reduced to a set of differential equations by means of the differential approximation [9] and the assumption of an appropriate mean value α of the frequency-dependent absorption coefficient. Thus in addition to the equations (1)–(3) we use the following approximate equations of the radiation field:

$$\operatorname{div} \mathbf{q}_R = 4\alpha[\sigma T^4 - \pi I_m], \tag{4}$$

$$\operatorname{grad} I_m = -\frac{3\alpha}{4\pi} \mathbf{q}_R. \tag{5}$$

Thereby the quantity

$$I_m = \frac{1}{4\pi} \int_0^{4\pi} \int_0^{\infty} I_\nu \, d\nu \, d\Omega \tag{6}$$

is the mean of the spectral intensity I_ν with respect to the radiation frequency ν and the space angle Ω, and σ is the Stefan–Boltzmann constant.

The use of one of the more refined versions of the differential approximations, as discussed for example in [4], is delayed until Section 3.

The nonlinear differential equations (1)–(5) are now linearized by assuming that there are only small perturbations of the stratified equilibrium state

$$\psi = \psi_0(y)+\psi'(x,y,t), \tag{7}$$

here ψ stands for any of the dependent variables, the subscript 0 indicates quantities in the undisturbed basic state and the superscript $'$ denotes the perturbation quantities.

Furthermore we adopt the Boussinesq-approximation, cf. e.g. [10], neglecting perturbations of density except as the buoyancy term in the momentum equation is concerned. It has been shown previously [2, 3] that this approximation is appropriate to describe thermoconvective waves in a liquid as well as in a gas, as long as longitudinal oscillations are not taken into consideration.

The approximations introduced above finally lead to the following set of linear differential equations:

$$\operatorname{div}\mathbf{w}' = 0, \tag{8}$$

$$\left(\frac{\partial}{\partial t}-\nu\Delta\right)\operatorname{rot}\mathbf{w}'+g\beta_0 T'_x = 0, \tag{9}$$

$$\left(\frac{\partial}{\partial t}-\kappa\Delta\right)T'+\gamma_0 v' = -\frac{1}{\rho_0 c_{p_0}}\operatorname{div}\mathbf{q}'_R, \tag{10}$$

$$(\Delta-3\alpha^2)\operatorname{div}\mathbf{q}'_R = 16\alpha\sigma T_0^{*3}\Delta T', \tag{11}$$

Δ denotes the Laplace operator and T_0^* the temperature in the undisturbed state at the reference plane $y = 0$, cf. Fig. 1; β is the coefficient of volumetric expansion, ν the kinematic viscosity, κ the thermal diffusivity, c_p the specific heat capacity. As all these coefficients are taken in the undisturbed, but stratified basic state, we assume that the relative changes of these coefficients due to the temperature distribution in y-direction are so small that we may use appropriate mean values in our calculation. The influence of a gradient of the viscosity, probably the most important deviation from the assumption of constant coefficients, has been considered in [2] showing no striking effect on the qualitative behaviour of thermoconvective waves.

As far as the coefficient γ_0 in equation (10) is concerned, a careful investigation is in order [3]. If the compressibility of the fluid in the stratified equilibrium state is not neglected γ_0 takes the form

$$\gamma_0 = \frac{\mathrm{d}T_0}{\mathrm{d}y}+\frac{g\beta_0 T_0^*}{c_{p_0}} = \frac{T_0}{c_{p_0}}\frac{\mathrm{d}s_0}{\mathrm{d}y}. \tag{12}$$

As $\gamma_0 < 0$ is a necessary condition for thermoconvective waves [1], not only a negative temperature gradient but rather a negative entropy gradient $\mathrm{d}s_0/\mathrm{d}y$ is required. This means that the stability criterion in an unbounded stratified fluid in the gravity field is not satisfied. Thermoconvective waves can therefore propagate only in a horizontal layer of finite thickness. As long as the Rayleigh-number (17) in such a layer remains subcritical, stability is guaranteed also at the large adverse temperature gradients that are desired to achieve a distinct weak damping of the thermoconvective waves. The absolute value of the negative temperature gradient, $|\mathrm{d}T_0/\mathrm{d}y|$, will then typically be large compared to the isentropic gradient, $g\beta_0 T_0^*/c_{p_0}$, so that for the purpose of the present investigation we may regard the coefficient γ_0, cf. (12), as approximately equal to the temperature gradient $\mathrm{d}T_0/\mathrm{d}y$.

Contrary to all the other coefficients in the system (8)–(11) we shall in general not be able to regard γ_0 as a constant, because a nonlinear temperature distribution in the basic state is just one of the especially important radiative effects, cf. Fig. 1. Before we can go into further details we have to define the parameters that characterize the radiation field. This is achieved by rewriting equation (11) in dimensionless form

$$(\bar{\Delta}-Bu^2)\overline{\operatorname{div}\mathbf{q}'_R} = H\,Bu^2\bar{\Delta}\bar{T}, \tag{13a}$$

with

$$\begin{aligned}\bar{T} &= \frac{T'}{T_0^*},\\ \overline{\operatorname{div}\mathbf{q}'_R} &= \frac{L_x^2}{\lambda T_0^*}\operatorname{div}\mathbf{q}'_R,\\ \bar{\Delta} &= L_x^2\Delta,\\ Bu &= \sqrt{3}\alpha L_x,\\ H &= \frac{16\sigma T_0^{*3}}{3\alpha\lambda} = \frac{\kappa_R}{\kappa}.\end{aligned} \tag{13b}$$

The Bouguer-number Bu measures the optical thickness on a distance L_x that will later on be set equal to the wave length of the weakly damped thermoconvective wave, cf. equation (18b).

The parameter H relates characteristic quantities of the radiation field to the conductivity λ. In the limit $Bu \to \infty$, equation (13a) shows that H may be regarded as the ratio between a fictitious radiant diffusivity for infinitely large optical thickness, κ_R, and the actual thermal diffusivity κ.

In the limit $Bu \to 0$, at a fixed value of the parameter H, the influence of radiation on the flow field vanishes altogether. A parameter to characterize the relative importance of radiation at arbitrary values of the Bouguer-number is defined by

$$G = \frac{H\,Bu^2}{1+Bu^2}, \tag{14}$$

G is equivalent to the parameter

$$\Gamma = \frac{Bu}{Bo(1+Bu^2)},$$

as it was introduced in [9], if we interpret the quantity

$$(H\,Bu)^{-1} = \frac{\sqrt{3}}{16}\frac{\lambda/L_x}{\sigma T_0^{*3}},$$

as a Boltzmann-number Bo.

If in the limit $Bu \to 0$ the parameter G instead of H is now held constant, then equation (13a) describes the limiting case of so-called dominating emission [2, 9].

Here the opportunity will be taken to introduce an improved differential approximation: If the definitions of Bu and H in (13b) are replaced by

$$Bu = \sqrt{3\alpha_P \alpha_R} L_x, \qquad H = \frac{16\sigma T_0^{*3}}{3\lambda\alpha_R}, \tag{15}$$

where α_P denotes the Planck mean and α_R the Rosseland mean of the absorption coefficient, cf. e.g. [11], then equation (13a) can be regarded as being formulated according to Traugott's modified differential approximation [12, 4]. This approximation yields, in the limits $Bu \to 0$ and $Bu \to \infty$, the same equations as would follow from the exact integro-differential-equations in these limits. Hence this method already provides a considerable increase of accuracy compared to the simple grey-gas approximation. A further discussion whether it would be adequate in our problem to use the linear Planck mean $\alpha_{L,P}$ [13] instead of α_P lies beyond the scope of this paper. More emphasis is laid on the question: at what temperatures already significant radiative effects can be expected. In Table 1 some typical data for different gases have been listed to show that in the considered problem radiation can attain the same importance as conduction even at remarkably low temperatures. Very high temperatures are only required in the case of air. (The necessary gas-properties were taken from [14, 15].)

Before solving now the system of equations (8)–(11) we must pay attention to the fact that the temperature gradient in the basic state is not constant. The wellknown solution for the nonlinear temperature profile in a conducting and radiating fluid layer, cf. e.g. [16], will not be reproduced here; we only want to discuss the main conclusions that are essential for our further calculations.

Obviously the deviation from a linear temperature distribution cannot be strong if the relative importance of radiation is only small, i.e. $G \ll 1$. On the other hand, if the optical thickness of the layer is large, $\alpha b \gg 1$, then the temperature gradient will be constant in the main part of the layer and the nonlinearity of the profile will manifest itself only in narrow regions near the walls of the layer. This means that in the cases

$G \ll 1$ with arbitrary values of the optical thickness (αb); and
$\alpha b \gg 1$ with arbitrary values of the parameter G,

the temperature gradient and hence the coefficient γ_0 in equation (10) can be regarded as constant. Then the solution of the system (8)–(11) may be found in the form of horizontally propagating waves with plane surfaces of constant phase.

In all the other cases the influence of radiation leads to typically S-shaped temperature profiles, cf. Fig. 1. The appropriate method to solve the equations (8)–(11) with a variable coefficient γ_0 will be discussed in Section 4.

3. INTERACTION BETWEEN THERMOCONVECTIVE WAVES AND RADIATION-INDUCED WAVES

We consider now the case where all the coefficients of the equations (8)–(11) are constant and assume the solutions to be of the form

$$\psi' = \Psi \cdot e^{i(kx - \omega t)}. \tag{16}$$

Table 1. A comparison of some typical data to show the relative importance of radiation

	$\sqrt{\alpha_P \alpha_R}$* [m^{-1}]	Bu†	H‡	G§
CO, 500 K	10.4	0.18	36.8	1.15
H_2O, 500 K	56.8	0.98	6.58	3.22
CO_2, 500 K	162.5	2.8	1.73	1.53
CO_2, 2000 K	14.4	0.25	502.5	29.5
Air, 2000 K	—	$1.3 \cdot 10^{-9}$	—	$5.5 \cdot 10^{-7}$‖

*Geometrical mean between Planck's and Rosseland's mean of the absorption coefficient; also: optical thickness of a layer of depth $b = 1$ m.

†$Bu = \sqrt{3\alpha_P \alpha_R} L_x$ with $L_x = 10^{-2}$ m, i.e. Bouguer-number at a typical value of the wave length.

‡$H = 16\sigma T_0^{*3}/3\lambda\alpha_R$, ratio between the fictitious radiant diffusivity of optically thick radiation and the actual thermal diffusivity.

§$G = HBu^2/(1+Bu^2)$, characteristic parameter for the relative importance of the radiative energy flux.

‖At 2000 K the value of α_P of air is still very small: $\alpha_P = 7.4 \cdot 10^{-8}\,m^{-1}$; at the correspondingly small optical thickness the quantities α_R and H are meaningless so that the following formulations have been used:

$$Bu = \sqrt{3}\alpha_P L_x, \qquad G = \frac{16\sigma T_0^{*3}}{\lambda} \alpha_P L_x^2, \text{ with } L_x = 10^{-2}\,m.$$

The amplitude Ψ of any of the perturbation quantities ψ' and the angular frequency ω are determined by the boundary conditions at the wall $x = 0$; the complex wave number k has to be calculated as a function of ω and the other relevant parameters.

Due to the horizontal walls bounding the fluid we actually would have to solve the problem of a two-dimensional wave propagation with an amplitude profile $\Psi(y)$ over the cross-section of the layer. But in [3] it has been shown that at large Rayleigh-numbers

$$Ra = \frac{g\beta_0|\gamma_0|b^4}{\kappa\nu} \gg 1, \tag{17}$$

the influence of the horizontal walls on the wave number is only of the order of $O(Ra^{-1/2})$. This means that for the purpose of a first order calculation with respect to the wave number k we may neglect the horizontal walls and thus the influence of the amplitude profile $\Psi(y)$ on the wave number. If we introduce wave solutions of the form (16) with constant Ψ into the equations (8)–(11) we obtain a characteristic equation (dispersion relation) of the following dimensionless form:

$$\bar{k}^2\left[\left(-i\varepsilon+\frac{1}{\sqrt{Pr}}\bar{k}^2\right)(-i\varepsilon+\sqrt{Pr}\,\bar{k}^2)-\bar{\gamma}_0\right] + Bu^2\left[\left(-i\varepsilon+\frac{1+H}{\sqrt{Pr}}\right)(-i\varepsilon+\sqrt{Pr}\,\bar{k}^2)-\bar{\gamma}_0\right] = 0, \tag{18a}$$

with

$$\bar{k} = kL_x,\quad L_x = \left[\frac{\kappa\nu}{-g\beta_0\gamma_0^*}\right]^{1/4},$$
$$\bar{\gamma}_0 = \frac{\gamma_0}{\gamma_0^*},\quad \varepsilon = \frac{\omega}{N},\quad N = \sqrt{-g\beta_0\gamma_0^*}. \tag{18b}$$

H and Bu are given according to (13b) or (15); $\bar{\gamma}_0$ denotes the ratio between the actual temperature gradient γ_0, and the temperature gradient without radiative effects γ_0^*. According to the assumptions of this section $\bar{\gamma}_0$ is constant but not necessarily equal to unity. Both γ_0 and γ_0^* must be negative for thermoconvective waves to exist, cf. [3]. This had to be taken into account when defining the characteristic length in x-direction, L_x, and the parameter N that corresponds to the buoyancy frequency or Brunt–Väisälä frequency, cf. [17, 18]. The parameter ε, the ratio between the angular frequency ω and the buoyancy frequency N, is the main parameter to characterize one-dimensional thermoconvective waves in a purely conducting medium, cf. [3].

The dispersion relation for the case of negligible radiation follows from (18) in either of the limits $Bu \to 0$ or $H \to 0$, and at $\bar{\gamma}_0 = 1$. If in addition the limit $\varepsilon \to 0$ is considered one obtains

$$\bar{k}^4 - 1 = 0, \tag{19}$$

with the two roots

$$\text{(I)}\quad \bar{k} = \pm 1,\qquad \text{(II)}\quad \bar{k} = \pm i, \tag{20}$$

corresponding to the weakly damped and strongly damped mode of thermoconvective waves respectively. Comparing now (20), (18b) and (17) we can derive

$$\lim_{\varepsilon\to 0}\left(\frac{k_r^{-1}}{b}\right)^2 = \frac{1}{\sqrt{Ra}}, \tag{21}$$

with k_r being the real wave number (2π/wave-length) of the weakly damped mode of the thermoconvective waves. This relationship between the wave-length of thermoconvective waves and the Rayleigh-number has been the basis of the perturbation analysis in [3] and will again be of importance in Section 5.

Due to the additional radiant energy transport the dispersion relation is now no longer a biquadratic, but a bicubic algebraic equation, cf. (18a), with three different solutions for the wave number $\bar{k}$. In addition to the two thermoconvective waves we obtain a so-called radiation-induced wave. As was already mentioned in the introduction radiation-induced waves have so far been studied in connection with a sound wave [7] and a thermal wave [6]. In both cases an interaction between a single classical wave and a single radiation-induced wave was observed. In analogy one might have expected now that together with two thermoconvective waves also two waves of the radiation-induced type would appear. But already equation (18a) shows that there exists no radiant counterpart for each of the two thermoconvective waves but that again only one single radiation-induced wave is obtained.

To achieve a better understanding of the phenomenon let us consider the limit $H \to 0$ in equation (18a) which leads to

$$(\bar{k}^2+Bu^2)\left[\left(-i\varepsilon+\frac{1}{\sqrt{Pr}}\bar{k}^2\right) \times (-i\varepsilon+\sqrt{Pr}\,\bar{k}^2)-\bar{\gamma}_0\right] = 0. \tag{22}$$

The second expression between brackets corresponds to thermoconvective waves in a merely conducting medium and the first expression

$$\bar{k}^2 + Bu^2 = 0, \tag{23}$$

can be associated with the radiation-induced wave. This degeneration of the wave number to only an imaginary part can also be followed directly from equation (13a) where in the limit $H \to 0$ the left hand side becomes equal to zero with equation (23) as the characteristic equation. We can conclude that in a radiating medium according to equation (13a) or (11) the temperature perturbations of any arbitrary wave system will stimulate one and only one radiation-induced wave. Furthermore, only by means of the coupling with other waves a radiation-induced wave with finite wave length and phase velocity is possible. The properties of the radiation-induced wave depend therefore very much on the special type of wave it is coupled with. Nevertheless in all cases studied so far (sound waves, thermal waves, thermoconvective waves) the wave numbers of the different kinds of radiation-induced waves approximate

$$k^2 = -3\alpha^2, \tag{24}$$

in the limit of weak radiation.

Besides it should be mentioned that in the limit of infinitely large optical thickness, $Bu \to \infty$, equation (18a) reduces to a fourth order dispersion relation of the thermoconvective type except for the fact that then the *sum* of the fictitious radiative diffusivity κ_R and the thermal diffusivity κ appears [2]. Again the effects of optically thick radiation are equivalent to an enhancement of thermal conductivity.

Now the actual numerical solutions of equation (18a) will be discussed. $(\bar{k}_r)^{-1}$, the reciprocal real part of the wave number (characterizing the wave length) and $\bar{k}_i$, the imaginary part of $\bar{k}$ (corresponding to the damping) have been calculated as functions of Bu^2, the square of the Bouguer-number, with ε and H as parameters, cf. the definitions (13b) and (18b) respectively.

Figure 2 shows the solutions at $\varepsilon = 0.1$ and with the radiation–conduction parameter $H = 0.01$. For simplicity we choose $Pr = 1$ and $\bar{\gamma}_0 = 1$ in the following: let us consider at first the damping constant $\bar{k}_i$ at small values of Bu^2; we easily identify the solution (I) with $\bar{k}_i = O(\varepsilon)$ as the weakly damped mode of thermoconvective waves, the solution (II) with $\bar{k}_i = O(1)$ as the strongly damped mode of thermoconvective waves and the solution (III) with $\bar{k}_i = O(Bu)$ as the radiation-induced wave, cf. equation (23). If the parameter H were chosen exactly equal to zero ($H = 0.0$), the three solutions would continue in the form of straight lines into the region of Bouguer-numbers $Bu > 1$. At small but finite values of H, as in Fig. 2, we observe however that the solutions (II) and (III) exchange their physical meanings. If the Bouguer-number is increased continuously from values $Bu < 1$ to $Bu > 1$, then the solution of the formerly strongly damped thermoconvective wave (II) continues as the solution of the radiation-induced wave and vice versa. The notation is understood according to Table 2.

The analogous changing of parts between solution (II) and (III) occurs for the wave length (see the diagram at the top of Fig. 2). In the limit $H \to 0$ (infinitely weak radiation) the solution $\bar{k}_r^{-1}$ of the radiation-induced wave goes to infinity, whereas the

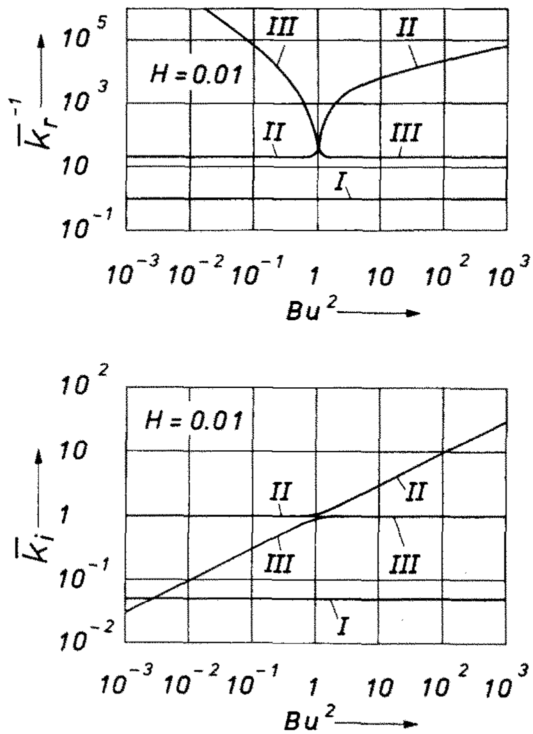

FIG. 2. Reciprocal real wave number $(\bar{k}_r)^{-1}$ and damping $\bar{k}_i$ for thermoconvective waves and a radiation induced wave, notation according to Table 2, with the frequency parameter $\varepsilon = 0.1$ and the radiation–conduction parameter $H = 0.01$ (weak radiation).

weakly and strongly damped thermoconvective waves adopt the values $\bar{k}_r^{-1} = 1$ and $\bar{k}_r^{-1} = O(\varepsilon^{-1})$ respectively.

Quite a similar exchange of roles between two solutions has already been observed in the case of interacting thermal and radiation-induced waves [6]. There, the phenomenon is limited to values of the radiation–conduction parameter $H > 1$; at $H = 1$ a singularity appears due to the fact that the two solutions of the complex wave number coincide and form a double root of the characteristic equation. As thermoconvective waves are concerned the analogous singularity is transferred to the limit $H = 0$ at $\varepsilon \to 0$. Increasing the parameter H from values $H \ll 1$ until $H \gg 1$ one obtains a continuous transition in the shape of the solutions from the type of Fig. 2 to the type $H \gg 1$ of Fig. 3. We notice that the

Table 2

Solution No.	$Bu < 1$	$Bu > 1$
I	weakly damped mode of thermoconvective waves	weakly damped mode of thermoconvective waves
II	strongly damped mode of thermoconvective waves	radiation-induced wave
III	radiation-induced wave	strongly damped mode of thermoconvective waves

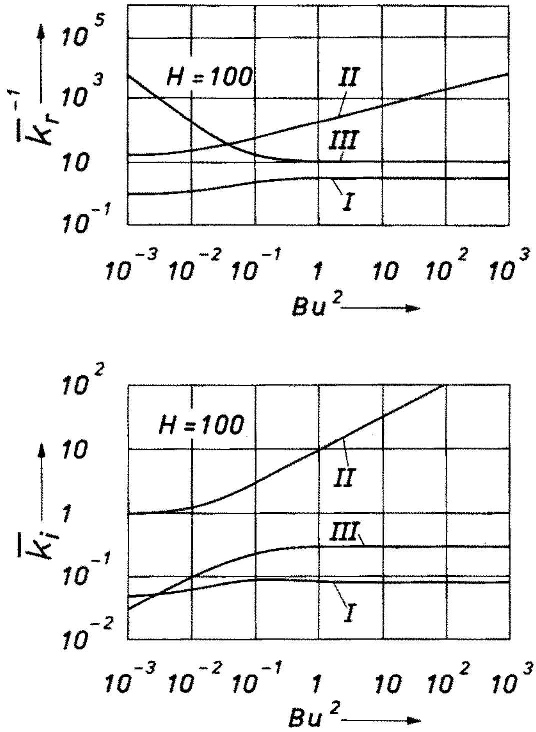

FIG. 3. Reciprocal real wave number $(\bar{k}_r)^{-1}$ and damping $\bar{k}_i$ at $\varepsilon = 0.1$ and $H = 100$ (strong radiation).

increasing relative importance of radiation influences mainly the solutions II and III whereas solution I, the weakly damped mode of thermoconvective waves, is only slightly modified even by strong radiation. This interesting result will be of importance in the analysis of the next section.

After the study of the wave numbers we have now to find out what relative magnitudes the amplitudes of the three simultaneously propagating waves will be. We seek those amplitudes that are induced at $x = 0$ according to two different boundary conditions.

(a) The wall $x = 0$ is impermeable with respect to radiation and its temperature changes according to a harmonic law. We further assume that the wall does not oscillate in either direction so that the sum of the velocity perturbations vanishes at $x = 0$. (At $x > 0$ the temperature perturbations induce also vertical translatorical oscillations v' by means of the buoyancy mechanism contained in thermoconvective waves.) The corresponding boundary conditions can be formulated as follows:

$$\left.\begin{aligned} T' &= \theta_0 \,\mathrm{e}^{-i\omega t} \\ v' &= 0 \\ q'_R &= 0 \end{aligned}\right\} \text{ in } \quad x = 0. \tag{25}$$

(b) Secondly we consider waves that are induced by an external radiation flux $q_e(t)$ that changes periodically with time. In analogy to [6] we assume that the fluid where the waves propagate is adjacent to a radiatively transparent and non-conducting medium. Under these assumptions the boundary conditions can be written as

$$\left.\begin{aligned} T'_x &= 0 \\ v' &= 0 \\ q'_R = q_e(t) &= Q_0 \,\mathrm{e}^{-i\omega t} \end{aligned}\right\} \text{ in } \quad x = 0. \tag{26}$$

The three equations for the amplitudes θ_j of the two thermoconvective waves and the radiation-induced wave that follow from the boundary conditions (25) or (26) together with the equations (8)–(11) and (16) are written down in full length in [2] but are omitted here. We only want to present the numerical results for a conduction–radiation parameter $H = 1$. Further results for $H = 0.1$ (weak radiation) and $H = 10$ (strong radiation) can be found in [2].

Figure 4 shows the absolute values (vector sum of real and imaginary part) of the temperature amplitudes according to the boundary conditions (25) of case (a). The solutions $|\theta_j|$ are obtained as functions of the wave numbers $\bar{k}_j$, the solutions of the dispersion relation (18a), so that once again an exchange of roles between the solutions II and III, cf.

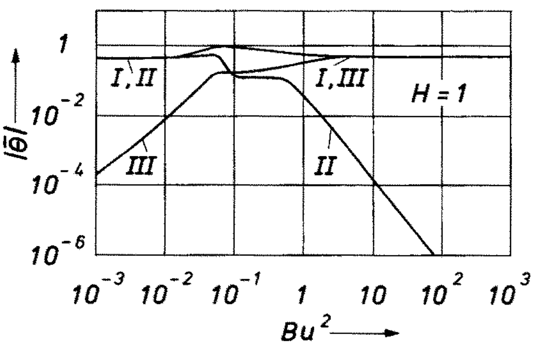

FIG. 4. Temperature amplitudes of thermoconvective waves and a radiation-induced wave according to a periodically changing wall temperature, boundary conditions (25), with $\bar{\theta}_j = \theta_j/\theta_0$ (j = I, II, III), and at $\varepsilon = 0.1$ and $H = 1$.

Table 2, can be observed. The amplitude $|\theta|$ of the radiation-induced wave is generally smaller than the amplitudes of the thermoconvective waves, except at $Bu = O(1)$ where the three amplitudes are of the same order of magnitude. Note, however, that this comparison concerns the maxima of the wave amplitudes that are in fact only present immediately at the wall $x = 0$; to quantify the disturbances at a certain distance x from the vertical wall one has to take into account also the different damping constants of the three waves, cf. Figs. 2 and 3, and [2].

In Fig. 5 the solutions for the temperature amplitudes induced by an external radiation flux (case b) are shown. Although the magnitudes of the three amplitudes do not differ from each other as much as in case (a), the exchange of roles between solutions II and III still appears. Contrary to case (a) the radiation field has now a strong influence also on the amplitude of the weakly damped thermoconvective wave (solution I). The results show that also a periodically changing

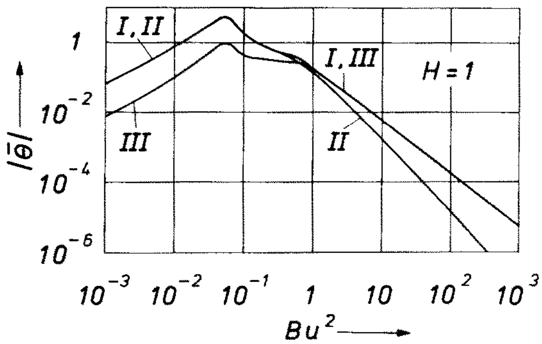

FIG. 5. Temperature amplitude according to periodically changing external radiation flux; boundary conditions (26), with $\bar{\theta}_j = \theta_j \sigma T_0^{*3}/Q_0$ (j = I, II, III), and at $\varepsilon = 0.1$ and $H = 1$.

radiation flux can give rise to thermoconvective waves simultaneously propagated with a radiation-induced wave. The amplitudes of the three waves, without account of the damping, are approximately of the same order of magnitude.

4. APPLICATION OF RAY METHODS TO THE CASE OF A NONLINEAR TEMPERATURE PROFILE

After the study of the interaction between thermoconvective waves and a radiation-induced wave by means of the plane wave model we turn now to the second important radiative effect, the nonlinear temperature distribution in the undisturbed basic state. Due to the varying temperature gradient the phase velocity is no longer constant and the surfaces of constant phase will no longer be plane. We have to show now whether we can replace the plane wave model by the methods of the ray theory (also called "geometrical acoustics" [19] or "geometrical optics" [20,21].)

The ray theory can only be applied if the characteristic length for changes of the phase velocity is large compared to a wave length. Considering especially a horizontal layer with a nonlinear temperature profile as in Fig. 1 we can regard the thickness b to be the characteristic length of significant changes of the basic state. The wave length of the weakly damped thermoconvective wave is small compared to the thickness b if we choose the value of the Rayleigh-number only a little smaller than the critical Rayleigh-number, cf. equation (21). This relationship remains valid also in the presence of radiation, because we have shown in the previous section that the wave number of the weakly damped thermoconvective wave retains its order of magnitude even at strong radiation. Under these circumstances it is justified to describe the propagation of the weakly damped thermoconvective wave in terms of the ray theory.

We consider again the two-dimensional problem and start from the equations (8)–(11) with a variable coefficient $\gamma_0(y)$ in the energy equation (10) corresponding to the nonlinear temperature distribution, cf. also (12). It is advantageous to reduce the system (8)–(11) to a single differential equation for the y-component of the velocity. Introducing further a formulation of the kind

$$\bar{v} = \varphi(\bar{x}, \bar{y})\,e^{-i\bar{t}}, \tag{27}$$

with

$$\bar{x} = x/b, \quad \bar{y} = y/b, \quad \bar{t} = \omega t,$$

finally leads to the following differential equation with respect to $\bar{x}$ and $\bar{y}$:

$$\bar{\Delta}[D_\nu D_\kappa \bar{\Delta}\varphi - \bar{\gamma}_0(\bar{y})\varphi_{\bar{x}\bar{x}}] - \tau^2[D_\nu D_R \bar{\Delta}\varphi - \bar{\gamma}_0(\bar{y})\varphi_{\bar{x}\bar{x}}] = 0, \tag{28a}$$

with

$$\begin{aligned} D_\nu &= -i\varepsilon - \sqrt{Pr}\,\frac{1}{K^2}\bar{\Delta}, \\ D_\kappa &= -i\varepsilon - \frac{1}{\sqrt{Pr}}\frac{1}{K^2}\bar{\Delta}, \\ D_R &= -i\varepsilon - \frac{1+H}{\sqrt{Pr}}\frac{1}{K^2}\bar{\Delta}, \\ \bar{\Delta} &= \frac{\partial^2}{\partial \bar{x}^2} + \frac{\partial^2}{\partial \bar{y}^2}, \\ \tau &= \sqrt{3}\alpha b, \\ K &= \left(\frac{-g\beta_0\gamma_0^*}{\kappa\nu}\right)^{1/4} b. \end{aligned} \tag{28b}$$

The parameter ε is again the ratio of frequency ω to N, cf. (18b), and in $\bar{\gamma}_0$ the variable temperature gradient $\gamma_0(y)$ is related to its constant counterpart γ_0^* in the case of vanishing radiation.

Together with (17), (18b) and (21) we identify the parameter K as the ratio between the thickness of the layer b and the reciprocal real wave number of the weakly damped thermoconvective wave in the limit $\varepsilon \to 0$. Thus we obtain

$$\frac{1}{K^2} = \frac{1}{\sqrt{Ra}} \ll 1, \tag{29}$$

at large Rayleigh-numbers that are necessary for weakly damped thermoconvective waves. With condition (29) satisfied we may find now solutions of equation (28a) by means of the formulation of the ray theory, cf. e.g. [20, p. 374]

$$\varphi = e^{iK\chi(\bar{x},\bar{y})} \sum_{m=0}^{\infty} (iK)^{-m}\phi_m(\bar{x},\bar{y}). \tag{30}$$

With the introduction of the eikonal $\chi(\bar{x},\bar{y})$ the phase $\mathscr{P}$ takes now the form

$$\mathscr{P} = K\chi(\bar{x},\bar{y}) - \omega t, \tag{31}$$

and the lines of constant phase in the $\bar{x},\bar{y}$-plane are given by $\chi = \text{const}$. Hence the phase velocity C can be written as

$$C = \frac{-\mathscr{P}_t}{\sqrt{(\nabla\mathscr{P})^2}} = \frac{\omega}{\frac{K}{b}\sqrt{(\nabla\chi)^2}}, \tag{32}$$

with

$$(\bar{\nabla}\chi)^2 = \chi_{\bar{x}}^2 + \chi_{\bar{y}}^2,$$

cf. [20, p. 241]. For the amplitude, a formulation in terms of a regular expansion for large values of the parameter K is assumed, cf. (30). When this expansion in the form (30) is introduced into equation (28a), then the lowest order terms with respect to K^{-1} result in a partial differential equation for χ, the eikonal equation. Herein further simplifications are possible. At first we can exclude the case of large optical thickness because the temperature gradient would then be nearly constant and there would have been no need to replace the plane wave model by ray theory methods. If we further exclude the case of dominating radiation and assume

$$\frac{H\tau^2}{1+\tau^2} = O(1) \ll K^2, \tag{33}$$

then any radiative influence in the eikonal equation vanishes except as the variable temperature gradient $\bar{\gamma}_0(\bar{y})$ is concerned

$$(\bar{\nabla}\chi)^2[-i\varepsilon+\sqrt{Pr}(\bar{\nabla}\chi)^2]\left[-i\varepsilon+\frac{1}{\sqrt{Pr}}(\bar{\nabla}\chi)^2\right] - \bar{\gamma}_0(\bar{y})\chi_{\bar{x}}^2 = 0. \tag{34}$$

Now exactly those terms are dropped that give rise to the radiation-induced wave in the plane wave model. This is due to the fact that the wave length of the radiation-induced wave is large compared to that of the weakly damped thermoconvective wave (Figs. 2 and 3) and that the long-wave modes are ignored in the context of the ray theory approximation.

In analogy to the dispersion relation we adopt now the asymptotic expansion

$$\chi = \chi_0 + \varepsilon\chi_1 + \ldots, \quad (\varepsilon \ll 1), \tag{35}$$

for small values of the frequency parameter ε and obtain

$$[\chi_{0\bar{x}}^2 + \chi_{0\bar{y}}^2]^3 - \bar{\gamma}_0(\bar{y})\chi_{0\bar{x}}^2 = 0. \tag{36}$$

This nonlinear first order partial differential equation which is no longer complex as is the former eikonal equation (34) is solved by introducing a characteristic ray coordinate ξ. The rays that can be regarded as the lines along which the wave energy propagates are then given in the $\bar{x},\bar{y}$-plane as a function of ξ: $\mathbf{x} = \mathbf{x}(\xi)$. We omit the details that can be found in [2] and shall concentrate on the main results.

The geometry of the wave propagation is described by the angle ϑ_r between the ray tangent and the x-axis and the angle ϑ_p between the phase normal (identical with the direction at the phase velocity) and the x-axis, cf. Fig. 6. In the classical isotropic case, e.g. a sound wave propagating in a motionless medium, the two angles ϑ_r and ϑ_p are identical. As thermoconvective waves are concerned the solution of the eikonal equation yields

$$\tan\vartheta_r = \tan\vartheta_p \frac{3\cos^2\vartheta_p}{3\cos^2\vartheta_p - 1}. \tag{37}$$

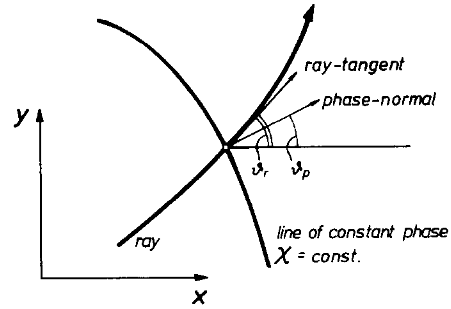

FIG. 6. The picture of a ray and a line of constant phase together with their inclination angles in the case of an anisotropic wave propagation in an inhomogeneous medium.

Except for the case $\vartheta_r = \vartheta_p = 0$ the angle of the ray direction ϑ_r is always larger than the angle of the phase normal ϑ_p, as it is indicated in Fig. 6. Such an anisotropy had to be expected already according to the fact that the differentiation in the eikonal equation (36) is not symmetrical with respect to $\bar{x}$ and $\bar{y}$. Furthermore we have to expect curvilinear rays as well as curvilinear lines of constant phase according to the variable coefficient $\bar{\gamma}_0(\bar{y})$ in the eikonal equation. One of the main results is a relationship that determines the curvature of the rays

$$\frac{d}{d\xi}(\tan\vartheta_p) = \chi_{0\bar{x}}\frac{d\bar{\gamma}_0}{d\bar{y}}, \tag{38}$$

with

$$\frac{d\chi_{0\bar{x}}}{d\xi} = 0.$$

As $\chi_{0\bar{x}}$ is constant along the rays equation (38) means the following: The angle of inclination of the phase velocity changes along the rays proportional to the local value of the second derivative of the temperature profile.

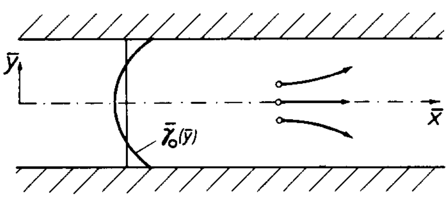

FIG. 7. Development of the ray curvature of thermoconvective waves at a distribution of the temperature gradient $\bar{\gamma}_0$ corresponding to a conducting and radiating fluid layer.

In a conducting and radiating layer with a distribution of the temperature gradient $\bar{\gamma}_0(\bar{y})$ as in Fig. 7 we have

$$\frac{d\bar{\gamma}_0}{d\bar{y}} > 0, \quad \text{at } \bar{y} > 0,$$

$$\frac{d\bar{\gamma}_0}{d\bar{y}} = 0, \quad \text{at } \bar{y} = 0,$$

$$\frac{d\bar{\gamma}_0}{d\bar{y}} < 0, \quad \text{at } \bar{y} < 0.$$

Hence initially horizontal rays are bent upwards in the upper half of the layer and downwards in the lower half of the layer (see Fig. 7). The ray in the middle of the layer as well as the corresponding phase normal remain always in the horizontal direction. The diverging behaviour of the rays leads to an attenuation of the amplitude in ray direction; we can also conclude that the phenomenon of a caustic will not occur at least as far as ray reflections from the walls are left out of consideration.

To obtain more accurate quantitative results for the amplitudes at a certain distance away from the oscillating source we would not only have to take into account the reflected rays but also the higher order solutions of the eikonal equation (34) with respect to ε as well as higher order solutions of equation (28a) with respect to K^{-1}, thus supplying so-called transport equations. We would then have to superpose the effects of attenuation of the amplitudes due to the diverging of neighbouring rays and the dissipative damping of the waves.

On the other hand we can state that the results of the first order ray-approximation have been derived irrespective of the fact that the nonlinear temperature distribution had originally been a consequence of radiation. The above results, especially the solutions (37) and (38) of the eikonal equation (36), are valid whether the variable temperature gradient is produced by a radiation field or any other nonlinear physical mechanism.

5. SUMMARY

The results of the investigation of thermo-convective waves under the influence of radiation can be summarized as follows:

(i) Radiation-induced waves can only adopt finite values of wave length and phase velocity when they are coupled with some other types of waves where temperature perturbations occur, e.g. thermoconvective waves. The properties of the radiation-induced waves depend therefore to a great extent on the properties of the waves they are interacting with. The example of thermoconvective waves has shown that the simultaneous propagation of different kinds of coupled waves in a radiation field does not give rise to different kinds of radiation-induced waves but yields only one single radiation-induced wave.

(ii) Under the assumption of one-dimensional plane harmonic waves a bicubic dispersion equation for the complex wave-number $\bar{k}$ is derived. The solutions for the reciprocal real wave number $(\bar{k}_r)^{-1}$ and the damping constant $\bar{k}_i$ are shown as functions of the Bouguer-number Bu (cf. Figs. 2 and 3). The solution that corresponds to the strongly damped mode of thermoconvective waves at $Bu \ll 1$ continues as the solution of the radiation-induced wave at $Bu \gg 1$ and vice versa. Such a changing of parts between two solutions that occurs here at any finite value of the conduction–radiation parameter H has been observed previously in the case of interacting ordinary thermal and radiation-induced waves, but there only at values $H > 1$. One can say that concerning the interaction with a radiation-induced wave the weakly damped mode of thermoconvective waves behaves like the ordinary thermal wave at $H \ll 1$ (weak radiation) whereas the strongly damped mode of thermoconvective waves behaves like the ordinary thermal wave at $H \gg 1$ (strong radiation). It is shown that the wave number of the weakly damped mode of thermoconvective waves, in contrary to the two other solutions, is only weakly modified by even a strong radiation field.

Furthermore the amplitudes of the three waves immediately at the wall where the perturbations are induced (i.e. at $x = 0$) are evaluated for two different boundary conditions (cf. Figs. 4 and 5). The results show that the greatest relative importance of the radiation-induced wave is approximately in the region $10^{-1} < Bu < 1$; there the amplitude as well as the damping reach values of about the same order of magnitude as for the thermoconvective waves.

(iii) The influence of radiation on thermoconvective waves manifests itself not only in the interaction with a radiation-induced wave, but also by way of a nonlinear temperature distribution in the undisturbed basic state. Only in the case of weak radiation or large optical thickness the nonlinearity of the temperature profile may be neglected so that a formulation in terms of plane waves is justified. In all the other cases one has to take account of a variable temperature gradient in the basic state and treat a system of differential equations with a variable coefficient. It is shown that the wave length of the weakly damped thermoconvective wave is small compared to the length of considerably large changes of the temperature gradient. Hence the condition for the application of the ray theory is satisfied.

The derived eikonal equation that describes the shape of the rays along which the wave energy propagates shows that we have to deal with an anisotropic wave propagation. The angle between the ray tangent and the horizontal x-axis is always larger than the angle between the phase normal and the x-axis. Furthermore the rays are bent towards increasing temperature gradients whereby the change in the direction of the phase velocity along the rays is proportional to the local second derivative of the temperature profile.

These results are independent of the fact that in our case the variable temperature gradient is due to radiation, and they apply whenever thermoconvective waves propagate in a stratified fluid with a nonlinear temperature distribution.

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REFERENCES

1. A. V. Luikov and B. M. Berkovsky, Thermoconvective waves, *Int. J. Heat Mass Transfer* **13**, 741–747 (1970).
2. H. Keck, Thermokonvektive Wellen unter Berücksichtigung von Kompressibilität, Wandeinflüssen und Wärmestrahlung, Diss. TU-Wien (1977).
3. H. Keck and W. Schneider, Thermoconvective waves in a compressible fluid, *Int. J. Heat Mass Transfer* **22**, 1501 (1979).
4. W. Schneider, Strahlungseffekte in Ein- und Mehrphasenströmungen, *Z. Angew. Math. Mech* **56**, T21–T36 (1976).
5. W. Koch, Radiation-convection interaction for a laterally bounded flow past a hot plate, *Int. J. Heat Mass Transfer* **17**, 915–931 (1974).
6. W. Schneider, Wärmewellen in einem leitenden und strahlenden Medium, *Z. Angew Math. Phys.* **22**, 517–532 (1971).
7. W. G. Vincenti and B. S. Baldwin, Effect of thermal radiation on the propagation of plane acoustic waves, *J. Fluid Mech.* **12**, 449–477 (1962).
8. W. G. Vincenti and S. C. Traugott, The coupling of radiative transfer and gas motion, *Ann. Rev. Fluid Mech.* **3**, 89–116 (1971).
9. W. Schneider, Grundlagen der Strahlungsgasdynamik, *Acta Mech.* **5**, 85–117 (1968).
10. E. A. Spiegel and G. Veronis, On the Boussinesq-approximation for a compressible fluid, *Astrophys. J.* **131**, 442–447 (1960).
11. W. G. Vincenti and C. H. Kruger, *Introduction to Physical Gas Dynamics*. Wiley, New York (1965).
12. S. C. Traugott, Radiative heat-flux potential for a non-grey gas, *AIAA Jl* **4**, 541–542 (1966).
13. A. C. Cogley, Differential approximation for radiative transfer in a non-grey gas near equilibrium, *AIAA Jl* **6**, 551–552 (1968).
14. C. L. Tien, Thermal radiation properties of gases, *Adv. Heat Transfer* **5**, 253–324 (1968).
15. W. M. Rohsenow and J. P. Hartnett (Eds.), *Handbook of Heat Transfer*. McGraw-Hill, New York (1973).
16. P. Niederdrenk, Thermokonvektive Instabilität einer strahlenden und wärmeleitenden Flüssigkeitsschicht, *ZAMM* **52**, 85–92 (1972).
17. J. S. Turner, *Buoyancy Effects in Fluids*. Cambridge University Press, Cambridge (1973).
18. E. E. Gossard and W. H. Hooke, *Waves in the Atmosphere*. Elsevier, Amsterdam (1975).
19. L. D. Landau and E. M. Lifschitz, *Lehrbuch der Theoretischen Physik. Bd. VI Hydrodynamik*. Akademie, Berlin (1971).
20. A. H. Nayfeh, *Perturbation Methods*. Wiley, New York (1973).
21. G. B. Whitham, *Linear and Nonlinear Waves*. Wiley, New York (1974).

ONDES THERMOCONVECTIVES DANS UN FLUIDE CONDUCTEUR ET RAYONNANT

Résumé—On considère l'influence du rayonnement thermique sur la propagation d'ondes thermoconvectives dans un fluide chauffé par le bas. Le champ radiatif est décrit au moyen de l'approximation différentielle. L'équation de dispersion qui est obtenue pour des ondes harmoniques, planes, à propagation horizontale, a trois solutions différentes:

En plus des deux modes d'ondes thermoconvectives, on obtient une onde de type induit par rayonnement avec des intéractions intéressantes. Si le champ radiatif conduit à une distribution de température fortement non linéaire dans l'état fondamental non perturbé, la propagation des ondes thermoconvectives faiblement amorties est décrite en terme de théorie du rayon.

THERMOKONVEKTIVE WELLEN IN EINEM LEITENDEN UND STRAHLENDEN FLUID

Zusammenfassung—Die Untersuchungen behandeln den Einfluß von Wärmestrahlung auf die Ausbreitung thermokonvektiver Wellen in einem von unten beheizten Fluid. Das Strahlungsfeld wird beschrieben durch die Gleichungen der Differentialapproximation. Für horizontal fortschreitende, ebene, harmonische Wellen wird eine Dispersionsgleichung abgeleitet, aus der man drei verschiedene Lösungen erhält: Neben den zwei thermokonvektiven Wellen tritt auch eine sogenannte strahlungsinduzierte Welle auf, wobei interessante Wechselwirkungen festzustellen sind. Führt das Strahlungsfeld zu einer stark nichtlinearen Temperaturverteilung im ruhenden Grundzustand, so wird die Ausbreitung der schwach gedämpften thermokonvektiven Welle mit Hilfe einer Strahltheorie beschrieben.

ТЕРМОКОНВЕКТИВНЫЕ ВОЛНЫ В ПРОВОДЯЩЕЙ И ИЗЛУЧАЮЩЕЙ ЖИДКОСТИ

Аннотация — Проведено исследование влияния теплового излучения на распространение термоконвективных волн в нагреваемой снизу жидкости. Поле излучения описывается с помощью дифференциального приближения. Дисперсионное уравнение, выведенное для описания горизонтально распространяющихся плоских синусоидальных волн, имеет три решения. В дополнение к двум модам термоконвективных волн получен ещё один тип волны, вызванной излучением, в результате чего можно наблюдать их взаимодействие. Если излучение создаёт существенно нелинейное распределение температуры в невозмущенном основном состоянии, тогда распространение слабо затухающих термоконвективных волн можно описать в приближении геометрической оптики.